

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
Second Semester B.Tech Degree Examination July 2021 (2019 scheme)

Course Code: MAT102

**Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND
TRANSFORMS**

(2019 Scheme)

FN Session

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

- 1 Find the unit tangent vector at a point $t_0 = \frac{\pi}{6}$ to the curve
 $\vec{r}(t) = \cos 3t \hat{i} + \sin 3t \hat{j} + 3t \hat{k}$. (3)
- 2 Find the directional derivative of the function $\phi = 3x^2y - y^3z^2$ at $(-1, -2, -1)$ in the direction of negative z axis. (3)
- 3 Using Green's Theorem evaluate $\int_C 2xy \, dx + (x^2 + x)dy$ where C is the unit circle in the positive direction. (3)
- 4 Determine whether the vector field $\vec{F}(x, y, z) = (y + z)\hat{i} - (xz^3)\hat{j} + (x^2 \sin y)\hat{k}$ is free of sources and sinks. (3)
- 5 Solve the initial value problem $y'' + y = 0$; $y(0) = 3, y'(0) = 1$. (3)
- 6 Find the Wronskian corresponding to the solution of $y'' - 3y' + 2y = 0$. (3)
- 7 Find the Laplace Transform of $\sin 3t \cos 2t$. (3)
- 8 Evaluate $L^{-1} \left[\frac{2}{(s+4)^3} \right]$. (3)
- 9 Find the Fourier cosine transform of the function $f(x) = \begin{cases} k, & 0 < x < a \\ 0, & x > a \end{cases}$ (3)
- 10 Express $f(x) = \begin{cases} \frac{1}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral. (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

- 11 a) Use a line integral to evaluate the work done by the force field $\vec{F} = xy^2\hat{i} + xy\hat{j}$ along the triangle with vertices $(0,0)$, $(2,1)$ and $(0,1)$ in the positive direction. (7)

- b) Use the given information to find the position and velocity vectors of the particle (7)
with acceleration $\vec{a}(t) = -\cos t \hat{i} - \sin t \hat{j}$, $\vec{v}(0) = \hat{i}$, $\vec{r}(0) = \hat{j}$
- 12 a) Evaluate $\int_C (xyz)dx - \cos(yz) dy$ where C is the straight line segment from (7)
(1, 1, 1) to (-2, 1, 3).
- b) Show that the vector field $\vec{F}(x, y) = \cos y \hat{i} - x \sin y \hat{j}$ is conservative and find (7)
 ϕ such that $\vec{F} = \nabla \phi$. Hence evaluate $\int_{(0,1)}^{(\pi,0)} \cos y dx - x \sin y dy$

Module-II

- 13 a) Using divergence theorem, evaluate $\iint_{\sigma} \vec{F} \cdot \hat{n} dS$, where (7)
 $\vec{F}(x, y, z) = (x^2 + y) \hat{i} + z^2 \hat{j} + (e^y - z) \hat{k}$ and σ is the surface of rectangular
cube bounded by the coordinate planes and the plane $x = 3, y = 1, z = 3$.
- b) Use Stoke's Theorem to evaluate the work done by the force field (7)
 $\vec{F}(x, y, z) = 3z \hat{i} + 4x \hat{j} + 2y \hat{k}$ over the boundary of the paraboloid
 $z = 4 - x^2 - y^2, z \geq 0$ with upward orientation.
- 14 a) Let σ be the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy -plane (7)
and σ is the oriented upwards. Find the flux of the vector field
 $\vec{F}(x, y, z) = x \hat{i} + y \hat{j} + z \hat{k}$ across σ .
- b) Find the mass of the lamina that is the portion of the plane $x + y + z = 2$ lying in (7)
the first octant where the density function of the surface is $\sigma = xz$.

Module-III

- 15 a) Solve using the method of undetermined coefficients: $y'' - 4y' + 4y = 4\sin^2 x$. (7)
b) Solve using the method of variation of parameters: $y'' + 4y = \sec 2x$. (7)
- 16 a) Solve using the method of undetermined coefficients: $y''' + 2y'' - y' - 2y = e^x$. (7)
b) Solve the initial value problem $x^2 y'' - 3xy' + 3y = 0, y(1) = 0, y'(1) = 1$. (7)

Module-IV

- 17 a) Using Laplace Transform solve $y'' + 4y' + 3y = e^{-t}, y(0) = 1, y'(0) = 1$. (7)
b) Using convolution theorem, find the inverse Laplace Transform of $\frac{18s}{(s^2+36)^2}$ (7)
- 18 a) Use Laplace Transform to solve $y'' + 3y' + 2y = u(t-1), y(0) = 0, y'(0) = 0$ (7)
b) Evaluate $L^{-1} \left[\frac{2s+1}{s^2+2s+5} \right]$ (7)

Module-V

- 19 a) Find the Fourier integral representation of the function $f(x) = \begin{cases} 1, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$. (7)
- b) Find the Fourier transform of $f(x) = e^{-|x|}, -\infty < x < \infty$ (7)
- 20 a) Represent $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ as Fourier Cosine Integral. (7)
- b) Find the Fourier sine transform of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ -1, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ (7)
